Generalized Momentum Control of the Spin-Stabilized Magnetospheric Multiscale (MMS) Formation

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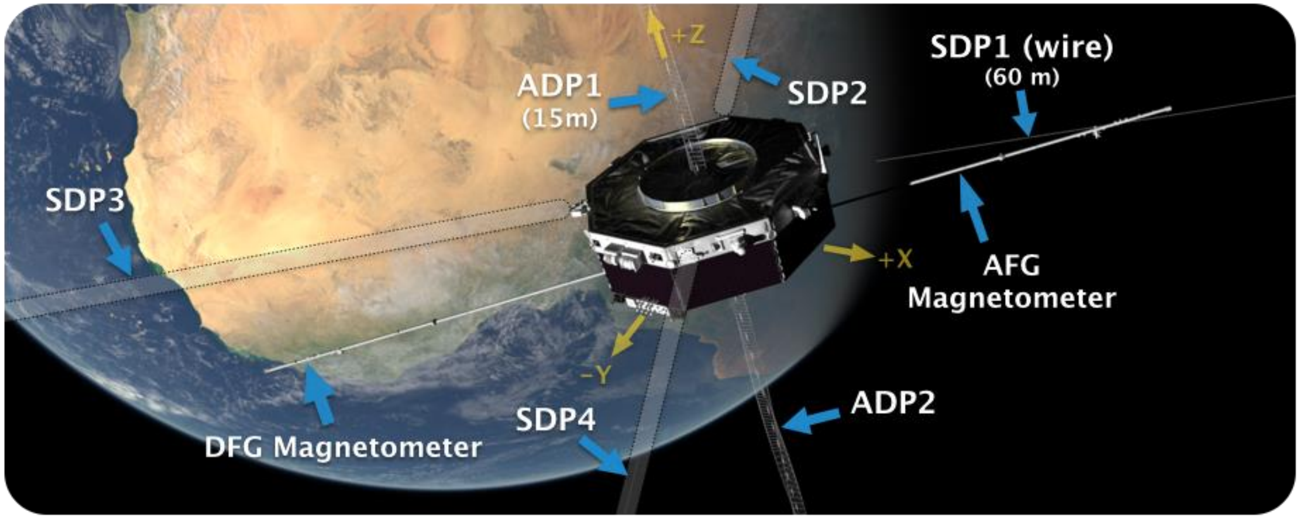
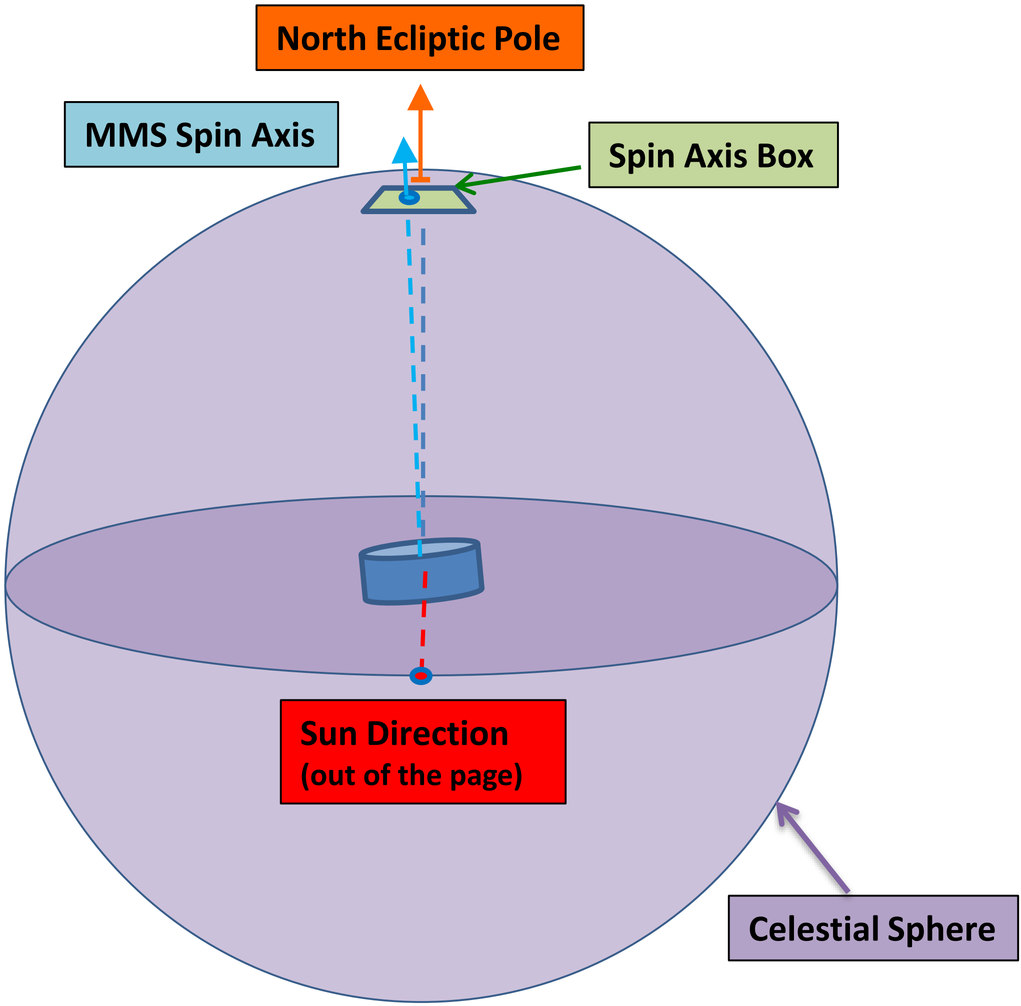
MMS Mission Overview



- **Mission Objective**
  - Heliophysics, Space-Based Magnetic Reconnection
- **Observatory Design**
  - Four Independent Spin-Stabilized Observatories in Tetrahedral Formation and Highly Elliptical Orbit
- **Instrument Design**
  - Instrument Suite Composed of 8 Deployable Booms

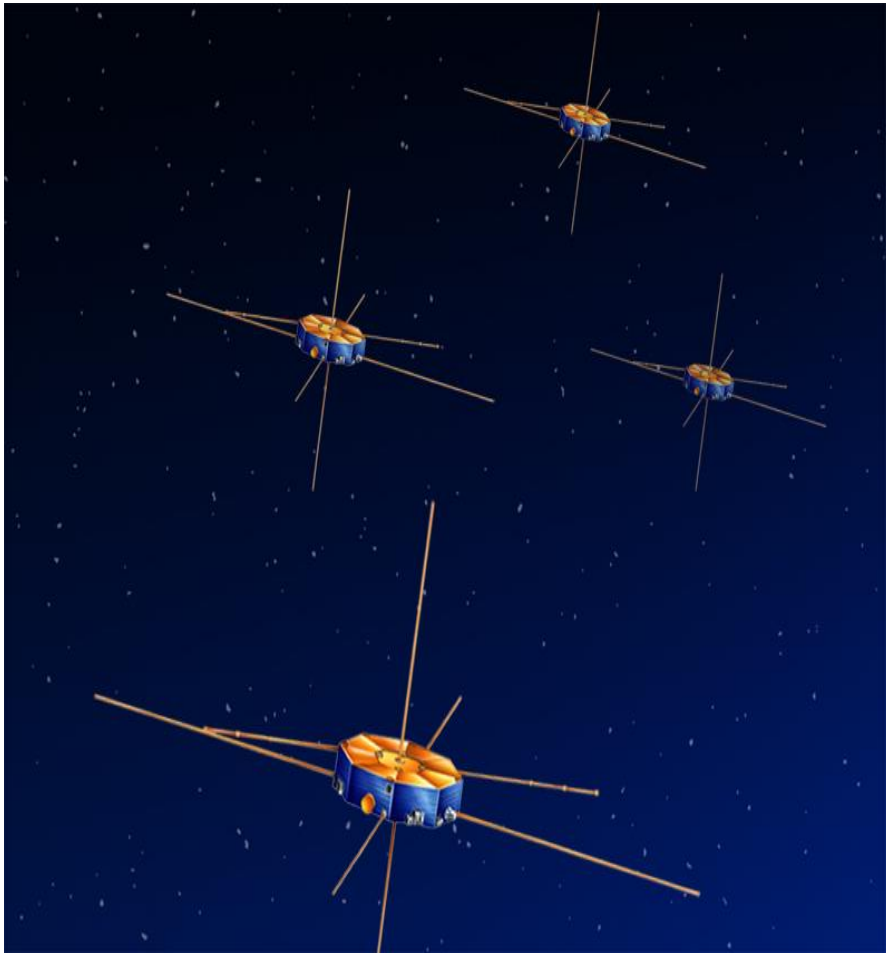
Angular Momentum Control Requirements

- **Spin-Axis (Pointing) Control:**
  - Thermally constrained 2x2 deg science box
- **Spin-Rate (Spin) Control:**
  - Nominal rate of 3 rev/min (RPM) to maintain SDP wire-boom tension
- **Transverse-Rate (Nutation) Damping:**
  - Minimize SDP motion and ensure spin-polarity for attitude sensors



Angular Momentum Control Design Challenges

- **Angular momentum control maneuvers required to keep spin-axis in science box**
- **Traditional approach uses de-coupled modes for pointing, spin, nutation**
- **Impractical for MMS**
  - Frequency and Number of maneuvers (Orbit Control, Pointing, Nutation, Spin, four observatories, every 2-4 weeks)
  - Difficult to implement de-coupled open-loop control with flexible wire booms
- **Desire a unified angular momentum controller**
  - Comprehensively control pointing, spin, and nutation



Angular Momentum Control Design Process

- **The MMS Angular Momentum Controller designed based on a controller developed by Reynolds and Creamer for the Reuven Ramaty High Energy Solar Spectroscopic Imager (RHESSI) mission.**
- **Reynolds-Creamer used the Lyapunov direct method for its formulation.**
- **MMS controller augments the Reynolds-Creamer baseline to include path-weighting.**

Lyapunov Direct Method

state vector

$$\mathbf{x} \to \mathbf{x}(t)$$

- For a non-linear dynamical system $\dot{\mathbf{x}} = f(\mathbf{x})$
  If a Lyapunov Function $V(\mathbf{x})$ exists, the system is stable about an equilibrium point
- Lyapunov Function if continuous and there is a neighborhood about the equilibrium point where for any $\mathbf{x}$

$V(\mathbf{x}) > 0$ about the origin

$V(\mathbf{x})$ Partial derivative continuous

$\dot{V}(\mathbf{x}) \leq 0$

Lyapunov Direct Method: Example

$\boldsymbol{\omega}$ angular rate

$\mathbf{I}$ Inertia matrix

$\boldsymbol{\tau}$ applied control torque

$[\,]^{\times}$ skew symmetric

- **For rigid-body spacecraft, dynamics represented by Euler's equations:**

$$\dot{\mathbf{x}} \rightarrow \dot{\boldsymbol{\omega}} = \mathbf{I}^{-1}\left(\boldsymbol{\tau} - [\boldsymbol{\omega}]^{\times}\mathbf{I}\boldsymbol{\omega}\right)$$

- **Lyapunov Function based on Rotational Kinetic Energy:**

$$V_{ex} = \frac{1}{2}\boldsymbol{\omega}^{\top}\mathbf{I}\boldsymbol{\omega} \longrightarrow$$

continuous positive definite

- **Lyapunov Derivative:**

$$\dot{V}_{ex} = \frac{1}{2}\boldsymbol{\omega}^{\top}\boldsymbol{\tau} \longrightarrow$$

pure rate damper if control torque applied opposite rate

General Lyapunov Control Design Methodology

1. Define a Lyapunov function in terms of the system states.
2. Differentiate the Lyapunov function
3. Substitute the system dynamics into the Lyapunov derivative.
4. Design a control law to ensure that the Lyapunov derivative is negative semi-definite.

Reynolds-Creamer Methodology

- **Reynolds-Creamer Lyapunov Function:**

$$V_{rc} = \frac{1}{2}\left(\mathbf{h} - \mathbf{h}_{pt}\right)^{\top}\left(\mathbf{h} - \mathbf{h}_{pt}\right) + \frac{1}{2}\mathbf{h}^{\top}\left(I_3\mathbf{I}^{-1} - \mathit{1}\right)\mathbf{h}$$

"pointing error" "spin error"

$\mathbf{h} = \mathbf{I}\boldsymbol{\omega}$ current angular momentum

$\underset{b\leftarrow i}{\mathcal{A}}$ inertial to body transformation

I_3 Principle major-axis inertia

$\mathbf{h}_{pt} = I_3\omega_0 \underset{b\leftarrow i}{\mathcal{A}}\,\hat{\mathbf{s}}_i$ *target* angular momentum

ω_0 *target* spin-rate

$\hat{\mathbf{s}}_i$ *target* spin-axis

$$\dot{V}_{rc} = \left(\boldsymbol{\omega} - \omega_0 \underset{b\leftarrow i}{\mathcal{A}}\,\hat{\mathbf{s}}_i\right)^{\top}\boldsymbol{\tau} \longrightarrow$$

align applied control torque in opposite direction of rate error: **unified** controller

Properties of Reynolds-Creamer Controller

- **Reynolds-Creamer is Path Unconstrained:**
  - For spinning spacecraft, nutation is induced to move spin-axis.
  - Spin-axis movement is unconstrained.
  - Unconstrained movement can cause high induced nutation.
  - Angular rates can be driven through zero.
- **Undesirable for MMS:**
  - SDP Wire Boom Motion
  - Spin-Polarity for Attitude Sensors

Path-Weighted Methodology

- **Path-Weighted Lyapunov Function:**

$$V_{pw} = \frac{k_{spin}}{2} \cdot \boldsymbol{\delta}\mathbf{h}_{pt}^{\top}\boldsymbol{\delta}\mathbf{h}_{pt} + \frac{1-k_{spin}}{2} \cdot \mathbf{h}_b^{\top}\boldsymbol{\delta}\mathbf{h}_b + \frac{1}{2}\mathbf{h}^{\top}\left(I_3\mathbf{I}^{-1} - \mathbb{1}\right)\mathbf{h}$$

"pointing error" "nutation error" "spin error"

$$\boldsymbol{\delta}\mathbf{h}_A \equiv \mathbf{h} - \mathbf{h}_A$$

$$\mathbf{h}_b = \omega_0 \hat{\mathbf{p}}_3$$

$\hat{\mathbf{p}}_3$ Major principal axis unit vector

k_{spin}

- **Range: [0,1]**
- **Weighting parameter dictating the level of induced nutation introduced during angular momentum control.**
- **"Encourages" the current spin-axis to remain in the region of the principal spin-axis during control.**

Path-Weighted Methodology

$$\dot{V}_{pw} = I_3 \left[\boldsymbol{\omega} - \omega_0 \left(k_{spin} \underset{b \leftarrow i}{\mathcal{A}} \hat{\mathbf{s}}_i + (1 - k_{spin}) \, \hat{\mathbf{p}}_3 \right) \right]^\top \boldsymbol{\tau}$$

$\longrightarrow$ **align applied control torque in opposite direction of rate error: unified controller**

MMS Implementation: Controller Error

Command Target Angular Velocity

$\hat{\mathbf{s}}_i$ Spin-Target Axis

ω_0 Spin-Target Magnitude

Calculate Controller Error

$$\boldsymbol{\delta}\omega_{pw} = \boldsymbol{\omega} - \omega_0 \left(k_{spin} \underset{b \leftarrow i}{\mathcal{A}} \hat{\mathbf{s}}_i + (1 - k_{spin}) \, \hat{\mathbf{p}}_3 \right)$$

$\boldsymbol{\omega}$ On-Board Estimate of Angular Rate

$\hat{\mathbf{p}}_3$ Ground Estimate of Principal Axis

MMS Implementation: Controller Actuation

MMS Actuators are Hydrazine Thrusters

eight 4-lbf radial

four 1-lbf axial

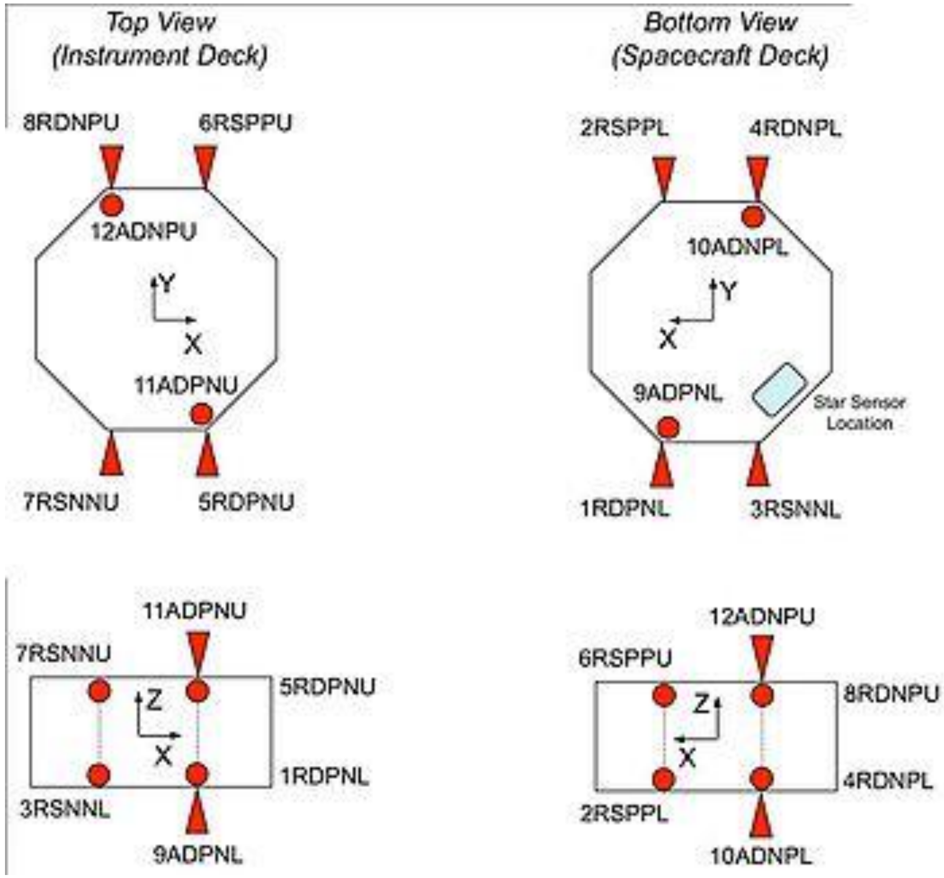
↓

14 pure moment pairs

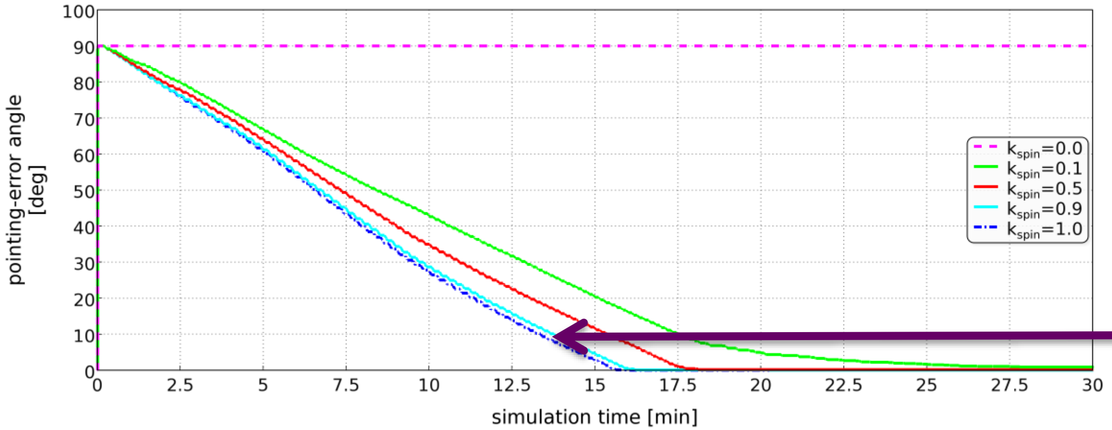
↓

If *k* ranges from 1-14

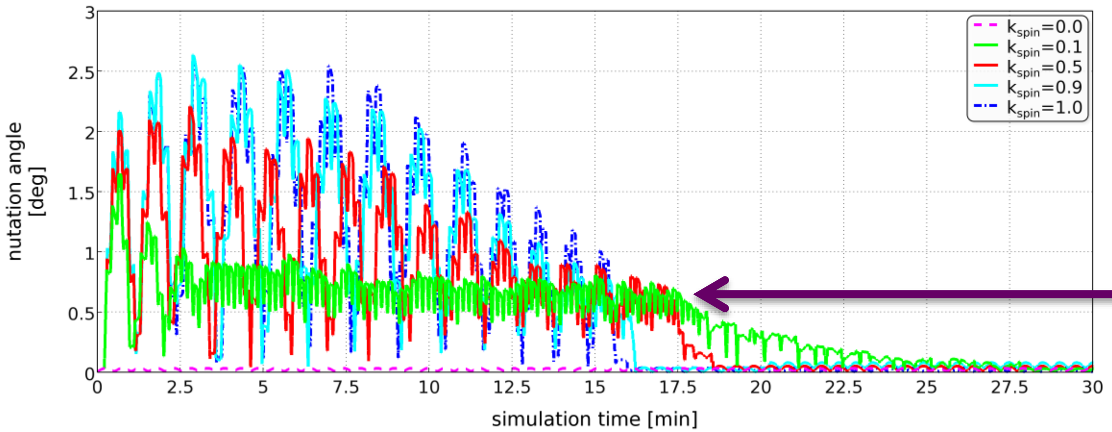
Determine which $\boldsymbol{\tau}_k$ is most negatively aligned with $\boldsymbol{\delta\omega}_{pw}$



Kspin Properties: Pointing Control

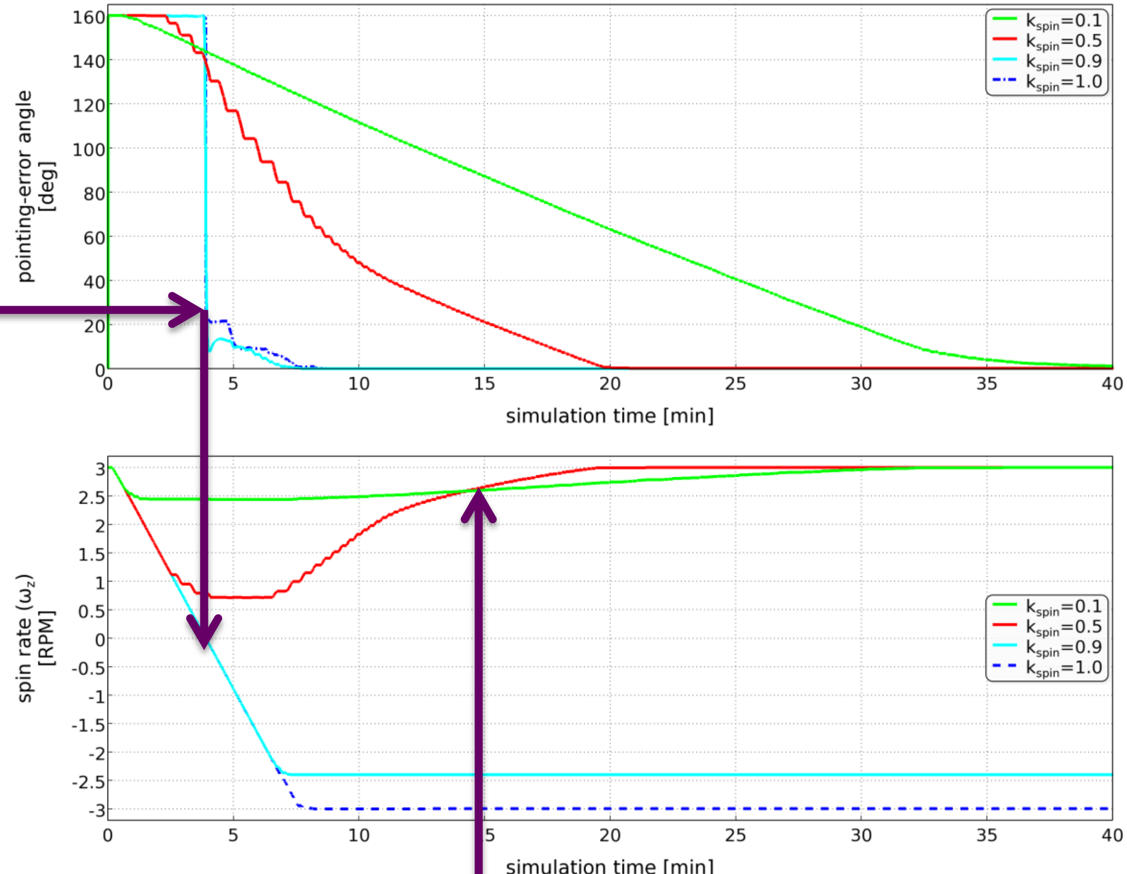


Higher gains cause the maneuver to complete in a



Lower gains limit the amount of induced nutation

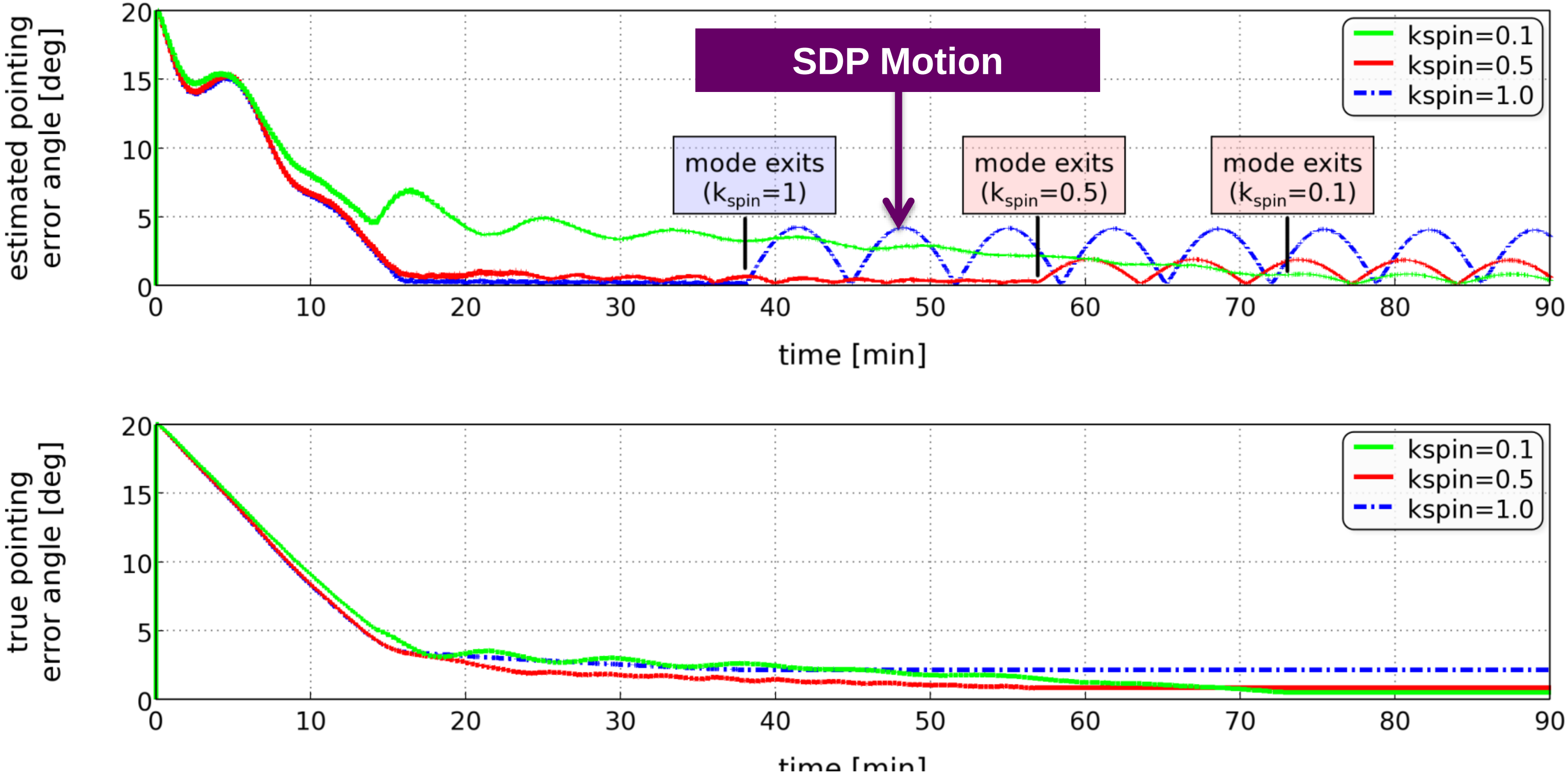
Kspin Properties: Spin Control



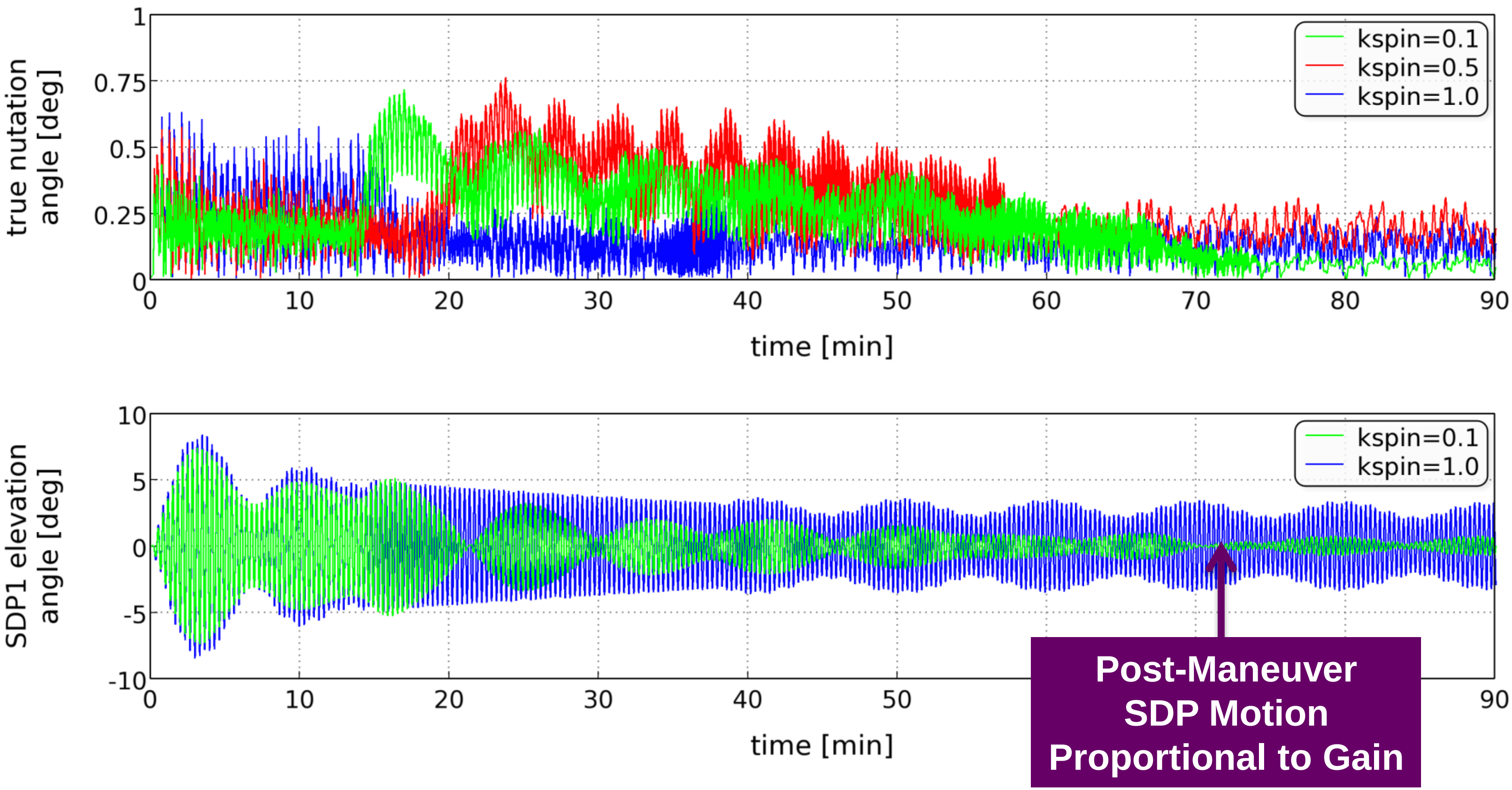
For higher gains, pointing is achieved, but spin is in opposite direction!

For lower gains, spin-axis direction is kept in neighborhood of major principal axis, spin is in same direction

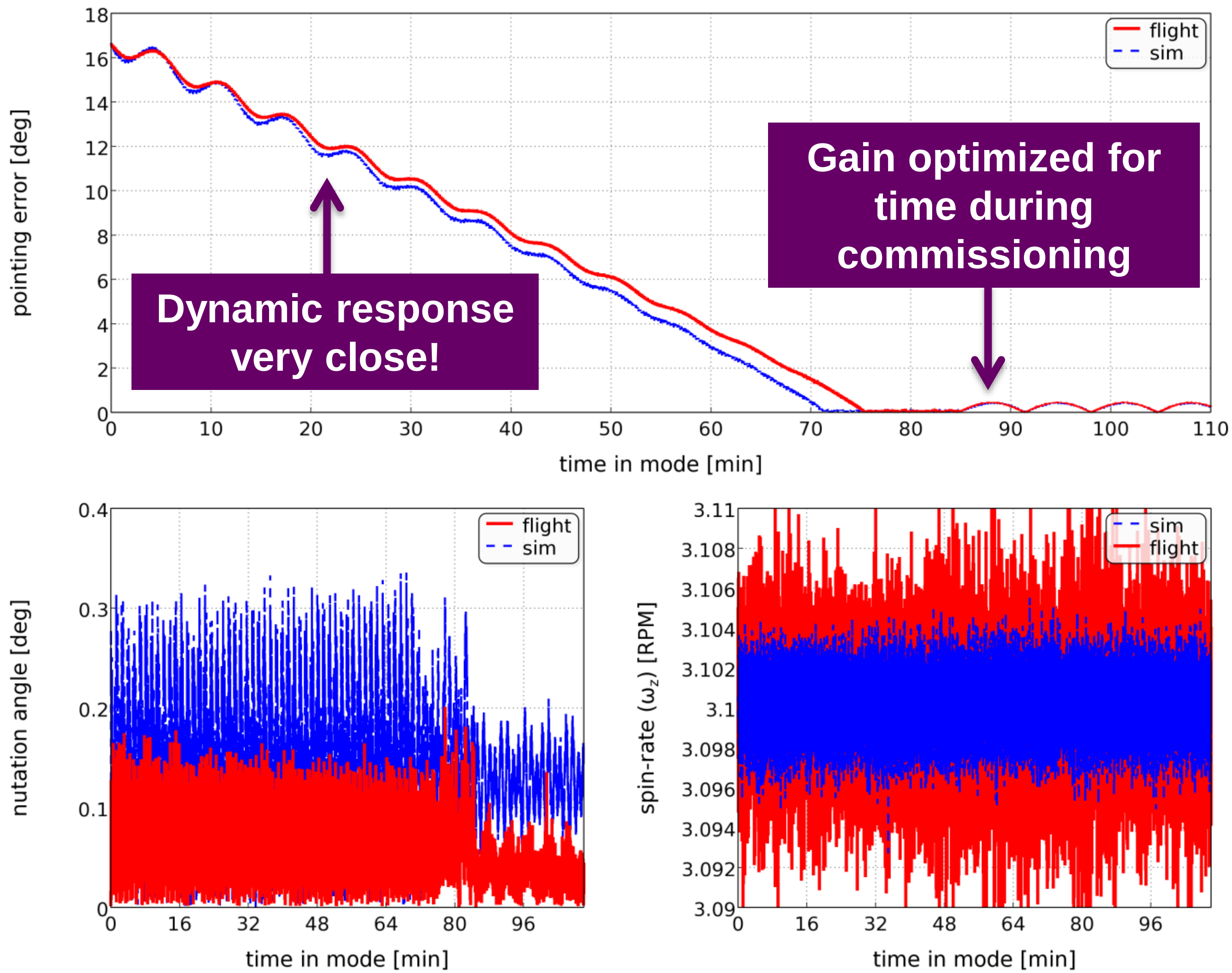
Kspin Properties: Flexible Body Dynamics



General characteristics for kspin design flexible dynamics



Flight Results



Flight Statistics

Maneuver (DOY)	**Observatory ID**	**Maneuver Duration** (min)	**Magnitude of Slew** (deg)	**Final Pointing Error Estimate** (deg)
GS-095 (167, 168)	1	40	2.49	0.24
	2	40	2.66	0.29
	3	20	0.87	0.25
	4	40	2.39	0.15
DH-116	1	40	2.18	0.06
FI-116 (188)	2	20	1.43	0.06
	3	20	1.07	0.16
	4	20	1.18	0.11
FI-119 (190)	1	-	-	-
	2	20	1.31	0.05
	3	20	1.21	0.15
	4	20	1.21	0.14

Summary

- **The MMS Angular Momentum controller is a unified controller.**
  - Can successfully control pointing, spin, and nutation for spin-stabilized observatories with flexible deployables.
- **Simple to use**
  - One control gain (kspin).
  - Commandable pointing axis and spin-rate.
  - Only requires knowledge of current angular rate and current major principal axis.
- **Can be used as a baseline for Angular Momentum control of future spin-stabilized missions.**

Questions?

Acknowledgements

References